Surname	Centre Number	Candidate Number
First name(s)		2

GCE A LEVEL

1300U30-1

S24-1300U30-1

TUESDAY, 4 JUNE 2024 – MORNING

MATHEMATICS – A2 unit 3
PURE MATHEMATICS B

2 hours 30 minutes

For Examiner's use only		
Question	**Maximum Mark**	**Mark Awarded**
1	**11**	
2	**11**	
3	**7**	
4	**6**	
5	**4**	
6	**13**	
7	**7**	
8	**7**	
9	**9**	
10	**14**	
11	**10**	
12	**6**	
13	**3**	
14	**7**	
15	**5**	
Total	**120**	

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ADDITIONAL MATERIALS

In addition to this examination paper, you will need:
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen. Do not use gel pen or correction fluid.

You may use a pencil for graphs and diagrams only.

Write your name, centre number and candidate number in the spaces at the top of this page.

Answer **all** questions.

Write your answers in the spaces provided in this booklet. If you run out of space, use the additional page(s) at the back of the booklet, taking care to number the question(s) correctly.

INFORMATION FOR CANDIDATES

The maximum mark for this paper is 120.

The number of marks is given in brackets at the end of each question or part-question.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

Answers without working may not gain full credit.

Unless the degree of accuracy is stated in the question, answers should be rounded appropriately.

You are reminded of the necessity for good English and orderly presentation in your answers.

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Reminder: Sufficient working must be shown to demonstrate the **mathematical** method employed.

1. The function f is given by

$$f(x) = \frac{25x+32}{(2x-5)(x+1)(x+2)}.$$

(a) Express $f(x)$ in terms of partial fractions. [4]

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(b) Show that $\int_1^2 f(x)\,dx = -\ln P$, where P is an integer whose value is to be found. [5]

(c) Show that the sign of $f(x)$ changes in the interval $x = 2$ to $x = 3$. Explain why the change of sign method fails to locate a root of the equation $f(x) = 0$ in this case. [2]

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2. (a) Find all values of θ in the range $0° < \theta < 360°$ satisfying

$$3\cot\theta + 4\operatorname{cosec}^2\theta = 5\ .$$

[5]

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(b) By writing $24\cos x - 7\sin x$ in the form $R\cos(x + \alpha)$, where R and α are constants with $R > 0$ and $0° < \alpha < 90°$, solve the equation

$$24\cos x - 7\sin x = 16$$

for values of x between 0° and 360°. [6]

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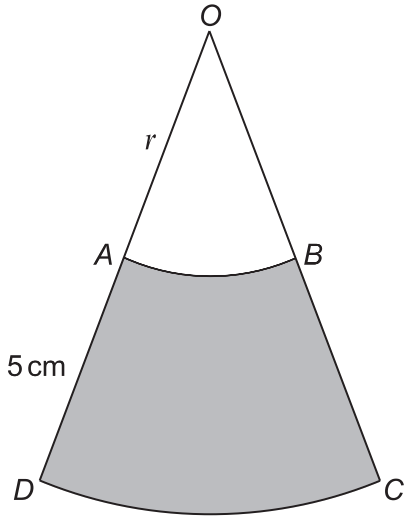
3. The diagram below shows a badge *ODC*. The shape *OAB* is a sector of a circle centre *O* and radius r cm. The shape *ODC* is a sector of a circle with the same centre *O*. The length *AD* is 5 cm and angle *AOB* is $\frac{\pi}{5}$ radians. The area of the shaded region, *ABCD*, is $\frac{13\pi}{2}$ cm^2.



(a) Determine the value of r. [4]

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(b) Calculate the perimeter of the shaded region. [3]

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4. A function f is given by $f(x) = |3x + 4|$.

(a) Sketch the graph of $y = f(x)$. Clearly label the coordinates of the point *A*, where the graph meets the x-axis, and the coordinates of the point *B*, where the graph cuts the y-axis. [3]

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(b) On a separate set of axes, sketch the graph of $y = \frac{1}{2}f(x) - 6$, where the points *A* and *B* are transformed to the points *A′* and *B′*.
Clearly label the coordinates of the points *A′* and *B′*. [3]

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5. Prove by contradiction the following proposition:

When x is real and positive, $x + \frac{81}{x} \geqslant 18$. [4]

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6. (a) Differentiate $\cos x$ from first principles. [5]

(b) Differentiate $e^{3x}\sin 4x$ with respect to x. [3]

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(c) Find $\int x^2 \sin 2x \, dx$. [5]

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 (1300U30-1) Turn over.

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7. Showing all your working, evaluate

(a) $\displaystyle\sum_{r=3}^{50}(4r+5)$, [4]

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(b) $\displaystyle\sum_{r=2}^{\infty}\left(540\times\left(\frac{1}{3}\right)^{r}\right)$. [3]

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8. The function f is defined by

$$f(x) = x^3 + 4x^2 - 3x - 1.$$

(a) Show that the equation $f(x) = 0$ has a root in the interval [0, 1]. [1]

(b) Using the Newton-Raphson method with $x_0 = 0 \cdot 8$,

(i) write down in full the decimal value of x_1 as given in your calculator,

(ii) determine the value of this root correct to six decimal places. [4]

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(c) Explain why the Newton-Raphson method does not work if $x_0 = \frac{1}{3}$. [2]

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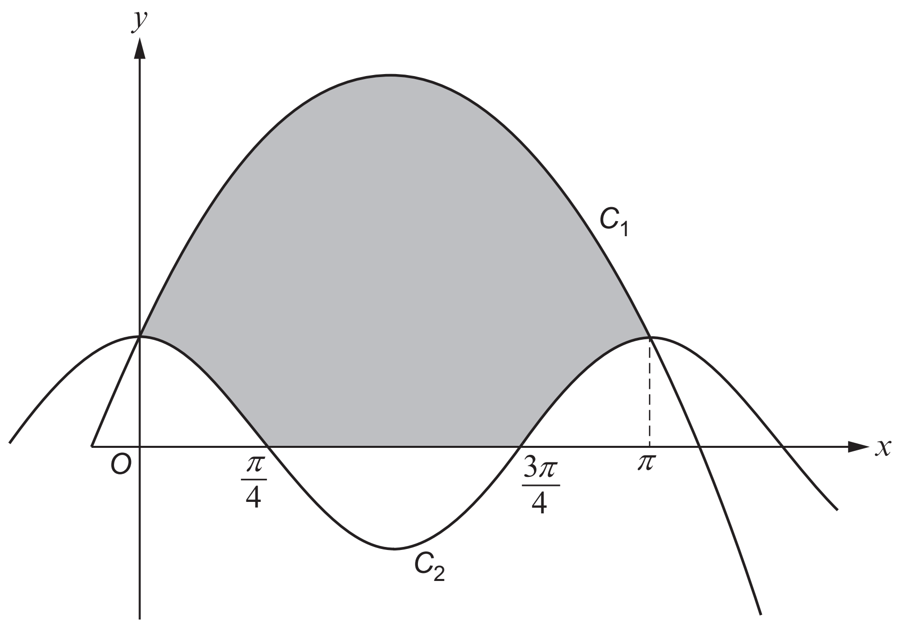
9. The diagram below shows a sketch of the curve C_1 with equation $y = -x^2 + \pi x + 1$ and a sketch of the curve C_2 with equation $y = \cos 2x$. The curves intersect at the points where $x = 0$ and $x = \pi$.



Calculate the area of the shaded region enclosed by C_1, C_2 and the x-axis. Give your answer in terms of π. [9]

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10. The function f has domain $[4,\infty)$ and is defined by

$$f(x)=\frac{2(3x+1)}{x^2-2x-3}+\frac{x}{x+1}.$$

(a) Show that $f(x)=\dfrac{x+2}{x-3}$. [4]

(b) Determine the range of $f(x)$. [2]

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(c) Find an expression for $f^{-1}(x)$ and write down the domain and range of f^{-1}. [4]

(d) Find the value of x when $f(x) = f^{-1}(x)$. [4]

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11. A curve is defined parametrically by

$$x = 2\theta + \sin 2\theta, \qquad y = 1 + \cos 2\theta.$$

(a) Show that the gradient of the curve at the point with parameter θ is $-\tan\theta$. [6]

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(b) Find the equation of the tangent to the curve at the point where $\theta = \dfrac{\pi}{4}$. [4]

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Turn over.

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12. (a) Given that θ is small, show that $2\cos\theta + \sin\theta - 1 \approx 1 + \theta - \theta^2$. [2]

(b) Hence, when θ is small, show that

$$\frac{1}{2\cos\theta + \sin\theta - 1} \approx 1 + a\theta + b\theta^2,$$

where a, b are constants to be found. [4]

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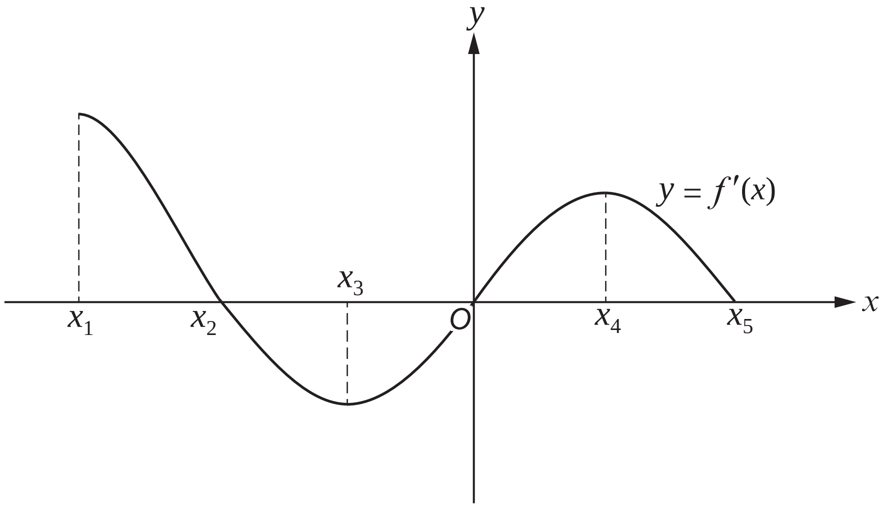
13. The diagram below shows a sketch of the graph of $y = f'(x)$ for the interval $[x_1, x_5]$.



(a) Find the interval on which $f(x)$ is both decreasing and convex. Give reasons for your answer. [2]

(b) Write down the x-coordinate of a point of inflection of the graph of $y = f(x)$. [1]

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14. (a) Given that $y = \dfrac{1 + \ln x}{x}$, show that $\dfrac{dy}{dx} = \dfrac{-\ln x}{x^2}$. [2]

(b) Hence, solve the differential equation

$$\frac{dx}{dt} = \frac{x^2 t}{\ln x},$$

given that $t = 3$ when $x = 1$.

Give your answer in the form $t^2 = g(x)$, where g is a function of x. [5]

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15. Robert wants to deposit £P into a savings account. He has a choice of two accounts.

- Account *A* offers an annual compound interest rate of 1%.
- Account *B* offers an interest rate of 5% for the first year and an annual compound interest rate of 0·6% for each subsequent year.

After n years, account *A* is more profitable than account *B*. Find the smallest value of n. [5]

END OF PAPER

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